STRESSES IN CYLINDERS EMBEDDED IN A STRESSED SOIL FIELD

by

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ABSTRACT

Solutions are developed for circumferential stresses in cylinders buried horizontally in a soil field which is stressed by the passage of a seismic wave. Although the problem is treated as qausi-static, the results with cylinder dimensions and soil modulus of practical significance are presumed to predict within 10%-15% the results which would be obtained for a more correct, but more involved dynamic formulation. The problem is of particular usefulness to designers of cylindrical storage tanks which are located underground in regions subjected to seismic disturbances.

The state of stress in the soil field and in the cylindrical tank is represented through the use of stress functions. Continuity of displacements across the interface surface between the soil and the cylinder, as well as the known tractions at the surface of the cylinders, provide the necessary boundary conditions to properly scale the terms in the stress functions.

Results of parametric studies are reported to demonstrate the influence on tank wall stresses of soil and tank physical properties, as well as the tank's geometric characteristics. A very simple upperbound solution, suitable for preliminary design, is presented for the average circumferential stress in the cylinder wall.

INTRODUCTION

Cylinders, such as fuel tanks, are often buried underground and must, therefore, resist stress induced by seismic waves which propagate through the soil in a direction perpendicular to the axis of the tank. The dimensions of a representative fuel tank is shown in Figure 1.

The determination of the dynamic response of buried cylindrical tanks subjected to the passage of stress waves in the surrounding soil has been studied extensively in recent years (1,2,3,4,5), and considerable information is now available on the subject. However, the solutions to the dynamic problem are rather complex and poorly suited for practical applications. Consequently, there appears to be no generally accepted procedure for the design of these tanks. Existing design procedures which are based on the assumption that the tank is subjected to the same strains as the soil field surrounding the tank (4), often result in unduly conservative stresses; other procedures based on published, theoretically correct, stress concentration factors (2,3) are applicable only for relatively thick tanks and relatively rigid soils. Therefore, a simple design procedure is needed which is applicable to thin, as well as to thick tanks, buried in either soft or stiff soils.

Due to the relative flexibility of the tank, the solutions to the applicable dynamic problems which have been studied (2,3,4) indicate that the maximum dynamically induced stresses in the cylindrical tank are only 10% to 15% larger than those which result from a corresponding static loading of the tank. Therefore, for most practical problems involving buried cylindrical tanks, the solution to the static problem can be used by the designer as a reasonable guide for the solution of the dynamic problem. Static solutions for stress concentrations around holes in infinite plates are available in (8). Some of these solutions are applicable to the problem at hand, although considerable numerical work is involved in evaluating the solution for a specific set of parameters; the numerical examples included in (8) do not include the range of parameters of interest for the buried tank problem.

The purpose of this study is to outline a method by which a relatively simple static solution can be obtained for the problem of a cylinder buried in a stressed soil field. Both the cylinder and the surrounding soil are assumed to be homogeneous and isotropic; the soil and the cylinder are assumed to be in intimate contact with no slip assumed at the interface between the two materials. The strain levels are assumed to be sufficiently low in the soil and the cylinder so that both materials behave elastically. To the extent that a static solution to the problem is applicable, there are no restrictions as to the relative stiffness of the cylinder and the surrounding soil, nor as to the radius to thickness ratio of the cylinder.

STATEMENT OF PROBLEM

A cylindrical tank is assumed to be buried in a soil field, as shown in Figure 1. The soil field is assumed to be uniformly stressed in a plane perpendicular to the axis of the cylinder as a result of the passage of a compression wave having a wave length relatively large compared to the diameter of the cylinder; this condition can be shown to exist for most seismic waves of practical interest. The basic stress condition in the soil is shown in Figure 2. The solution to the basic problem shown in Figure 2 can then be used to obtain more general solutions involving the stress fields induced by the passage of plane shear (S) waves or compression (P) waves in confined soil. The stress field in the free field soil, i.e., the soil in the absence of the tank, can be determined from the simple one dimensional wave equation which produces the result:

$$\sigma_A = \rho_s C_p \, V\max \quad \text{or} \tag{1}$$

$$\sigma_A = \rho_s C_s \, V\max$$

where

σ_A = resultant normal stress in the soil field;

ρ_s = mass density of the soil;

C_p, C_s = propagation velocity of a compression (P) wave or shear (S) wave, respectively;

Vmax = resultant maximum particle velocity of the soil field (determined from a study of the site response due to a specified seismic disturbance).

FORMULATION

The solution to the problem illustrated in Figure 2 can be reduced to that of two concentric cylinders by considering that an imaginary circular boundary exists in the soil at a radius much larger than that of the tank (i.e., $b/c \approx 0$). Stresses on this imaginary circular boundary of the soil are also shown in Figure 2. For convenience, the stresses on the boundary, $r = c$, are expressed as:

$$\sigma_r(c) = \sigma_{00} + \sigma_{02}\cos 2\theta; \quad \tau_{r\theta}(c) = -\sigma_{02}\sin 2\theta$$

where for this problem, $\sigma_{00} = \sigma_{02} = \frac{\sigma_A}{2}$. The stresses on the imaginary circular boundary of the soil are essentially unaffected by the presence of the cylindrical tank if the radius of the soil boundary is much larger than that of the tank. The stress in the soil field is assumed to be uniform with respect to the axial location along the length of the tank. Therefore, the problem is treated as a plane strain problem.

SOLUTION

The solution to the problem is obtained by first deriving the stresses, strains, and displacements in the soil field and in the cylinder region from appropriate stress functions. The stress functions chosen for this problem are:

$$\phi^i = (A_0^i \log r + B_0^i r^2 \log r + C_0^i r^2 + D_0^i) + \left(A_2^i r^2 + B_2^i r^4 + \frac{C_2^i}{r^2} + D_2^i\right)\cos 2\theta \tag{2}$$

$$i = 1, 2$$

where the superscript i = 1 refers to the soil region and i = 2 refers to the cylinder region.

Stresses derived from the stress function are given in (6) as:

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} \; ; \; \sigma_\theta = \frac{\partial^2 \phi}{\partial r^2} \; ; \; \tau_{r\theta} = \frac{-\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right) . \qquad (3)$$

Strains associated with the stresses are given by the well known (6) constitutive relationships associated with isotropic materials in plane strain.

Displacement may be derived through appropriate integration of the following kinematic relationships (6):

$$\varepsilon_r = \frac{\partial u}{\partial r} \; ; \; \varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} \; ; \; \gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \qquad (4)$$

The constants associated with the stress functions ϕ^i may be evaluated by imposing the required stress boundary conditions at r = a, r = b, and r = c (shown in Fig. 2), and the required displacement boundary condition at r = b. For this problem, full continuity was assumed between the cylinder and the soil at r = b so that radial stresses (σ_r), tangential stresses ($\tau_{r\theta}$), normal displacements (u), and tangential displacements (v) are equal for the two regions at their interface, r = b.

Although the 6 stress boundary conditions and the 2 displacement continuity conditions described in the previous paragraph are sufficient to evaluate the constants associated with the stress functions, prohibitively great numerical accuracy must be retained in the solution process to achieve meaningful results. An alternate solution scheme, which avoids some of the pitfalls of the previously described approach, is to first evaluate the constants of the stress functions in each region in terms of assumed interface stresses:

$$\sigma_r(b) = \overset{*}{\sigma_0} + \overset{*}{\sigma_2}\cos 2\theta \quad ; \; \tau_{r\theta}(b) = \overset{*}{\tau_2}\sin 2\theta$$

The solution for the constants are:

$$\begin{aligned}
&\frac{A_0^1}{b^2} = (-\sigma_{00} + \overset{*}{\sigma_0}) \quad ; && A_0^2 = \frac{-\alpha^2}{\alpha^2-1}\overset{*}{\sigma_0} \quad ; \\
&B_0^1 = D_0^1 = 0 \quad ; && B_0^2 = D_0^2 = 0 \quad ; \\
&2C_0^1 = \sigma_{00} \quad ; && 2C_0^2 = \overset{*}{\sigma_0}\frac{\alpha^2}{\alpha^2-1} \quad ; \\
&2A_2^1 = -\sigma_{02} \quad ; && 2A_2^2 = \frac{1}{\Delta}[-\overset{*}{\sigma_2}\alpha^2(\alpha^4+\alpha^2+2)+\overset{*}{\tau_2}(2\alpha^2)] \; ; \\
&B_2^1 = 0 \quad ; && 6B_2^2 b^2 = \frac{1}{\Delta}[\overset{*}{\sigma_2}\alpha^4(\alpha^2+3)-\overset{*}{\tau_2}\alpha^4(3-\alpha^2)] \; ; \\
&\frac{6C_2^1}{b^4} = -3\sigma_{02}+\overset{*}{\sigma_2}-2\overset{*}{\tau_2} \; ; && \frac{6C_2^2}{b^4} = \frac{1}{\Delta}[-\overset{*}{\sigma_2}(3\alpha^2+1)+\overset{*}{\tau_2}2] \; ; \\
&\frac{2D_2^1}{b^2} = 2\sigma_{02}-\overset{*}{\sigma_2}+\overset{*}{\tau_2} \; ; && \frac{2D_2^2}{b^2} = \frac{1}{\Delta}[\overset{*}{\sigma_2}(2\alpha^4+\alpha^2+1)-\overset{*}{\tau_2}(\alpha^2+1)] \; . \qquad (5)
\end{aligned}$$

where $\alpha = b/a$
and $\Delta = (\alpha^2-1)^3$

Displacements u and v may now be expressed in terms of the imposed outer boundary stresses and the interface stresses. Equations of displacement across the interface are then written, producing 2 algebraic equations in each of the 0th and 2nd harmonics which may be solved simultaneously for the interface stresses in terms of the applied outer boundary stresses. When the stress functions are evaluated in terms of the applied outer boundary stresses, all other stresses and displacements in the region of the soil and the cylinder are also known in terms of the applied stress σ_A.

RESULTS

The significant parameters in the problem under consideration are embodied in the non-dimensional parameters E_s/E_t, a/t, ν_s and ν_t. Numerical solutions have verified that the influence of Poisson's Ratio (ν_s, ν_t) is small, and will not be discussed further herein. The stresses of greatest interest are the tangential stresses σ_θ in the cylinder. Numerical results indicate that for almost all cylinders of interest the tangential stress distribution across the cylinder wall is almost linear. Therefore, for convenience, the tangential stress is considered in two parts:

$$\sigma_\theta(a) = \bar{\sigma}_\theta + \sigma_{\theta f} \tag{6}$$

where

$$\bar{\sigma}_{\theta max} = \left[\int_a^b \frac{\sigma_\theta}{t}\, dr\right]_{max} = \frac{(3\sigma_0^* - \sigma_2^* + 2\tau_2^*)}{3}\ \frac{b}{t}\ , \text{ a mean stress,} \tag{7}$$

and

$$\sigma_{\theta f} = |\sigma_\theta(a) - \bar{\sigma}_\theta|\ , \text{ a flexural stress.} \tag{8}$$

The influence of the modulus ratio E_s/E_t on the maximum flexural and mean stress in the cylinder wall is shown for representative values of a/t in Figs. 3, 4. Results from these figures, and others similar to them, indicate clearly that the maximum flexural stress component becomes a significant part of the total tangential stress in the tank wall only for relatively thick walls ($a/t < 150$) and for relatively weak soils ($E_s/E_t <$ .01). The maximum stress, σ_θ, in the tank wall is reduced as the modulus of the soil increases but is increased as the radius to thickness ratio of the tank increases.

The influence of the radius to thickness ratio, a/t, on resultant stresses in the tank wall is shown in Fig. 5 for a relatively weak soil. Results plotted in this figure indicate that the flexural stress component, $\sigma_{\theta f}$, of the tank's circumferential stress is small compared to the mean stress $\bar{\sigma}_\theta$ when a/t is large ($a/t > 150$), but becomes much larger than the mean stress for small values of a/t. The unusual peaking of the flexural stress component $\sigma_{\theta f}$. shown in Fig. 5 for weak soil, does not occur for stiffer soils. A representative plot for a stiffer soil is shown in Fig. 6.

DISCUSSION AND CONCLUSIONS

Since the tangential stress plays a dominant role in the design of the cylinder, several observations are made which lead to a simple approximate expression for this stress:

$$\left.\begin{array}{l}\sigma_0^* \simeq \sigma_2^* \\ \tau_2^* \simeq 2\sigma_2^* \\ \sigma_2^* \leq .6\sigma_A\end{array}\right\} \qquad \begin{array}{l}\text{for } a/t \geq 100 \\ \text{and } E_s/E_t > 10^{-4}\end{array}$$

Substitution of these approximate values in Eq. 7 produces the result that, for the conditions stated:

$$\left|\frac{\bar{\sigma}_\theta}{\sigma_A}\right| \approx 2.0 \left|\frac{\sigma_2^*}{\sigma_A}\right| \frac{b}{t} \leq 1.2\,\frac{b}{t} \tag{9}$$

Eq. 9 provides a reasonable estimate of maximum circumferential stress in the cylinder for weak soils and relatively low a/t ratios; for strong soils and/or relatively thin cylinders, Eq. 9 provides a conservative estimate of circumferential stress. The flexural stress component, $\sigma_{\theta f}$, of cylinder stress is relatively small for stiff soils and/or thin tanks. However, for thick tanks ($a/t < 150$) and weak soils ($E_s/E_t < .01$) the flexural stress component may be large compared to the cylinder's mean stress, $\bar{\sigma}_\theta$, and should be considered in the design of the cylinders.

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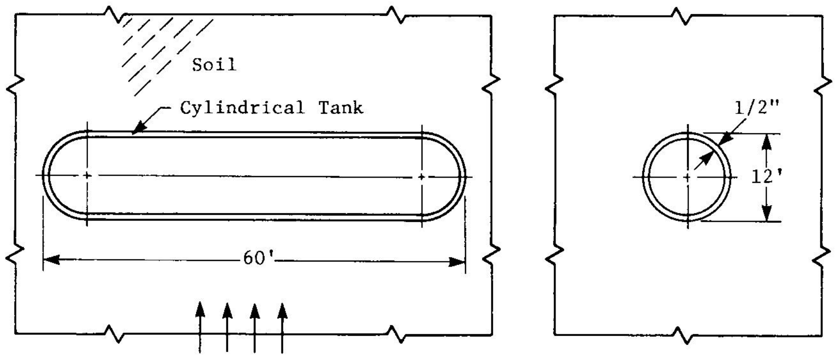

Figure 1. Cross Section of Representative Buried Cylindrical Tank

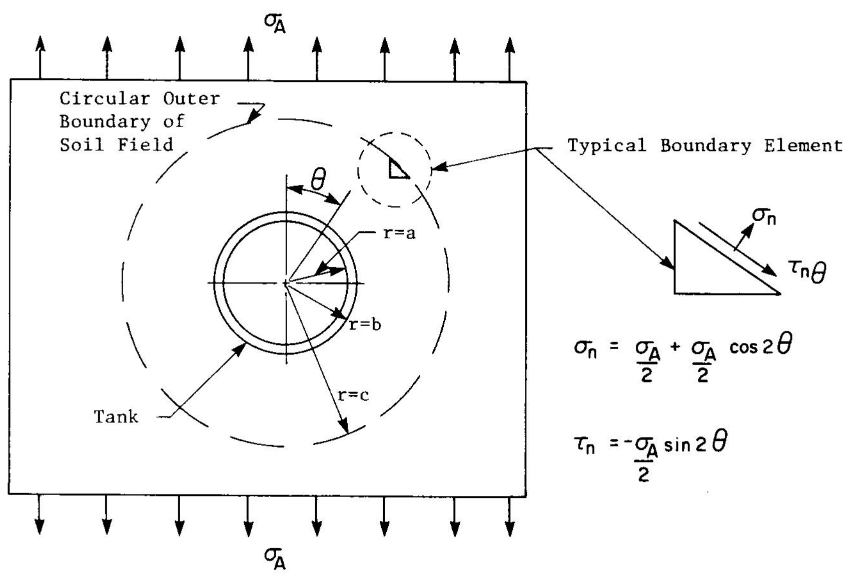

Figure 2. Assumed Stress Condition in Soil Field

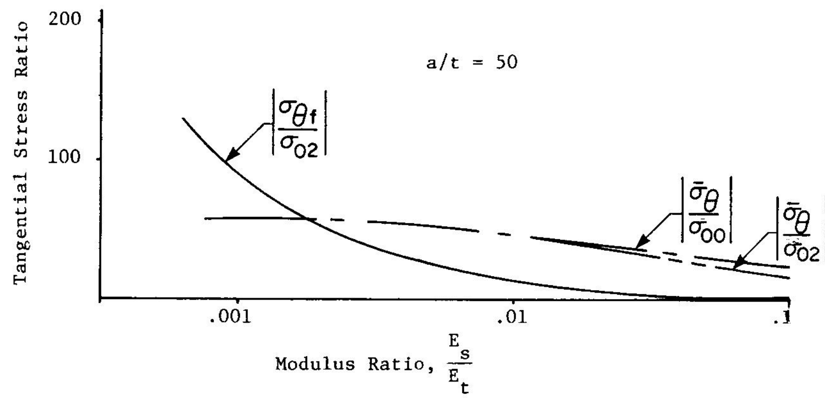

Figure 3. Tangential Stress Ratio vs. Modulus Ratio, for a/t = 50

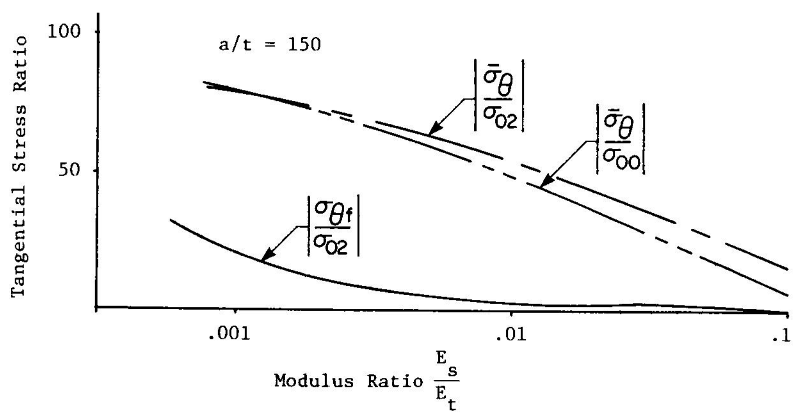

Figure 4. Tangential Stress Ratio vs. Modulus Ratio for a/t = 150

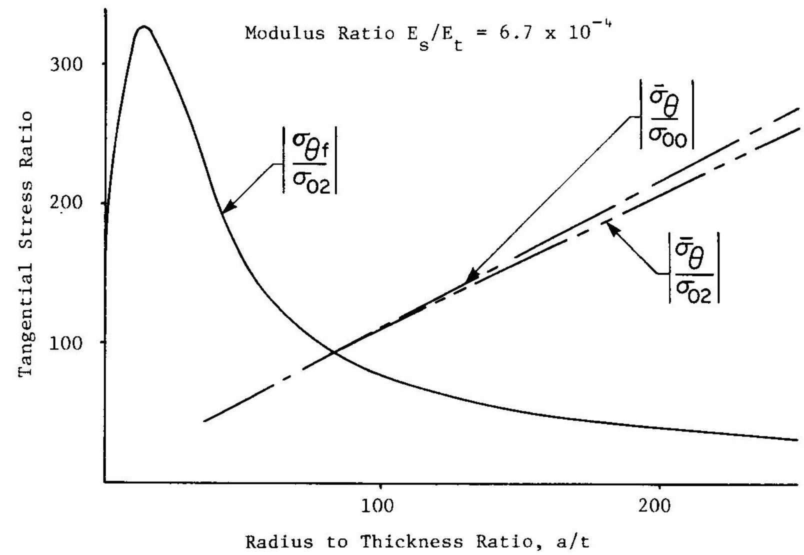

Figure 5. Tangential Stress Ratio vs. Radius to Thickness Ratio

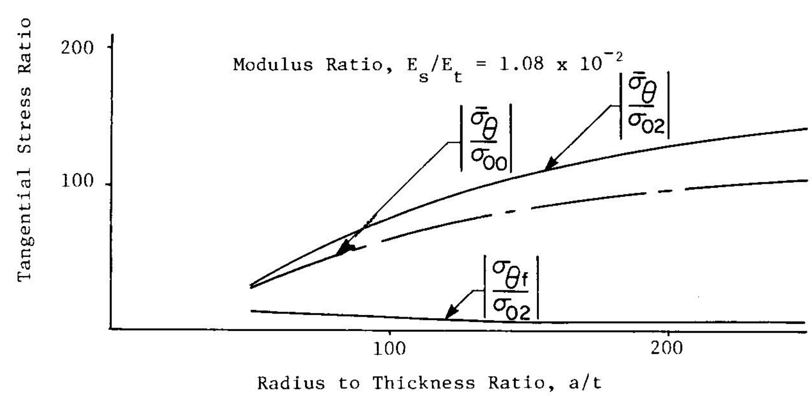

Figure 6. Tangential Stress Ratio vs. Radius to Thickness Ratio